On Blasius Flow in Non-Newtonian Fluid

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Blasius flow in non-Newtonian power-law fluids has been analytically studied by Acrivos et al. (1960), Luikov (1965) and Defrawi and Finlayson (1972). In the first, one finds an exact solution together with results obtained by applying Karman-Pohlhausen integral method. In the last, one finds results obtained in two ways on application of the same integral technique.

The purpose of this communication is to show that a perturbation technique applied to the exact Blasius equations obtained by Acrivos et al. (1960) leads to realistic results compared with other available ones.

Acrivos et al. (1960) assumed the velocity distribution in the form

$$\left.\begin{aligned} u &= U_\infty \; \phi(\eta), \\ \phi(\eta) &= 1.5 \; \eta - 0.5 \; \eta^3, \\ \eta &= y/\delta, \\ \phi(0) &= 0, \;\; \phi(1) = 1, \;\; \phi'(1) = 0. \end{aligned}\right\} \qquad (1)$$

Defrawi and Finlayson (1972) assumed the form

$$\phi(\eta) = 1 - (1 - \eta)^{(n+1)/n} \qquad (2)$$

The exact values are given by the equations

$$\left.\begin{aligned} n(n+1)\, f''' + f\,(f'')^{2-n} &= 0, \\ f(0) = f'(0) = 0, \;\; f'(\infty) &= 1. \end{aligned}\right\} \qquad (3)$$

The wall shear stress is proportional to

$$c(n) = [f''(0)]^n . \qquad (4)$$

Let us assume that the fluid is slightly non-Newtonian and that

$$\left.\begin{aligned} n &= 1 + \epsilon, \quad \epsilon \text{ being small}, \\ f &= f_0 + \epsilon f_1. \end{aligned}\right\} \qquad (5)$$

Substituting from (5) into (3) and equating the coefficients of different powers of ϵ to zero we get the equations

$$f_0''' + \tfrac{1}{2} f_0 f_0'' = 0,$$

$$f_1''' + \tfrac{1}{2} \{f_0 f_1'' + f_0'' f_1 - \tfrac{1}{2} f_0 f_0'' (3 + 2 \log f_0'')\} = 0,$$

$$f_0(0) = f_0'(0) = 0, \quad f_0'(\infty) = 1;$$

$$f_1(0) = f_1'(0) = 0, \quad f_1'(\infty) = 0.$$

The above equations have been solved numerically on an electronic computer. The solutions are

$$f_0''(0) = 0.332060, \quad f_1''(0) = 0.052821 \qquad (6)$$

Next we calculate the values of $c(n)$ by substituting from Equations (5) and (6) into Equation (4). Table 1 shows how they compare with the already obtained values.

NOTATION

$c(n)$ = shear stress coefficient defined by (4)
f = nondimensional stream function
n = parameter in constitutive equation for power-law fluids
u = velocity component in x direction
U_∞ = fluid velocity outside boundary layer
x = distance along the plate surface measured from the leading edge
y = distance normal to the surface

Greek Letters

δ = boundary layer thickness
η = nondimensional distance, y/δ
ϕ = nondimensional velocity defined in Equations (1) and (2)

LITERATURE CITED

Acrivos, A., M. J. Shah, and E. E. Peterson, "Momentum and Heat Transfer in Laminar Boundary-Layer Flows of Non-Newtonian Fluids Past External Surfaces," *AIChE J.*, **6**, 312 (1960).
Luikov, A. V., "Applications of the Methods of Thermodynamics of Irreversible Processes to the Investigation of Heat and Mass Transfer," *J. Eng. Phys.*, **9**, 287 (1965).
Defrawi, M. E., and B. A. Finlayson, "On the Use of the Integral Method for Flow of Power-Law Fluids," *AIChE J.*, **18**, 251 (1972).

TABLE 1.

n	Exact Acrivos et. al. (1960)	Integral method, Acrivos et al (1960)	Integral method, Defrawi and Finlayson (1972)	Roy
0.05	1.017	0.926	——	0.939
0.1	0.969	0.860	0.99	0.882
0.2	0.8725	0.75	0.89	0.779
0.3	0.7325	0.655	——	0.692
0.5	0.5755	0.518	0.61	0.553
1.0	0.33206	0.323	0.365	0.33206
1.5	0.2189	0.238	——	0.215
2.0	0.1612	0.169	——	0.148
2.5	0.1226	0.133	——	0.109
3.0	0.09706	0.109	——	0.084
4.0	0.06777	0.079	——	0.058
5.0	0.05111	0.061	——	0.048

Compared with the exact solutions, our results, though obtained on the assumption of small values of ϵ, are fairly good even for ϵ = 4.